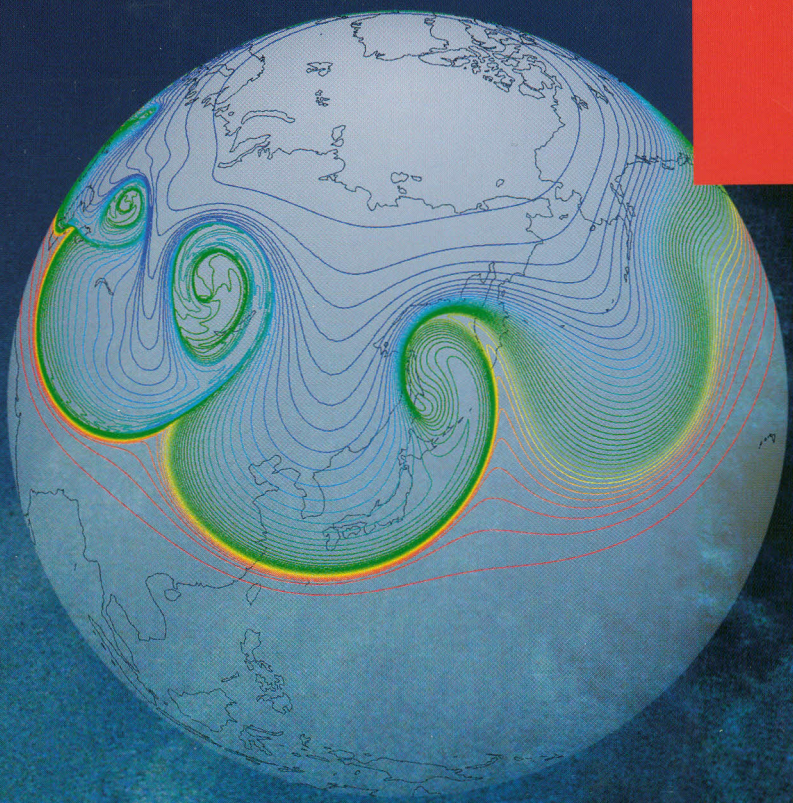
Atmospheric Circulation Dynamics and General Circulation Models



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Contents

Preface xv

List of figures xix

List of tables xxvii

Part I Principle ideas 1

1 Basic equations 4
1.1 Dry air 4
1.1.1 Equation of state and thermodynamic variables 4
1.1.2 Thermodynamic variables of the ideal gas 8
1.2 Conservation laws and basic equations 11
1.2.1 Conservation law and conservation of mass 11
1.2.2 Conservation of momentum 13
1.2.3 Conservation of energy 15
1.2.4 Entropy balance 18
1.2.5 Enthalpy balance and Bernoulli's theorem 19
1.3 Angulum momentum, vorticity, and divergence 20
1.3.1 Conservation of angular momentum 20
1.3.2 The circulation theorem 22
1.3.3 Vorticity equations 24
1.3.4 Potential vorticity 26
1.3.5 Divergence equation 29
References and suggested reading 31

2 Basic balances and stability 32
2.1 Hydrostatic balance 33
2.2 Geostrophic balance and thermal wind 36

2.3 Stability of hydrostatic balance 41
2.4 Stability of axisymmetric flows 44
2.5 Balances and stability of parallel flows on the f-plane 52
References and suggested reading 54

3 Approximations of equations 55
3.1 Boussinesq approximation 56
3.2 Quasi-geostrophic approximation 59
3.2.1 Scaling 59
3.2.2 Synoptic-scale quasi-geostrophic equations 60
3.2.3 Planetary-scale quasi-geostrophic equations 64
3.3 Primitive equations 65
3.3.1 The equations in spherical coordinates 65
3.3.2 Transformation of the vertical coordinate 67
3.3.3 Pressure coordinates 70
3.3.4 Sigma coordinates 71
3.3.5 Isentropic coordinates 73
3.4 Shallow water equations 76
3.5 Appendix: Derivation of a generalized pressure gradient 78
References and suggested reading 79

4 Waves 80
4.1 Wave theory 80
4.2 Sound waves 84
4.3 Gravity waves 87
4.3.1 Dispersion relation 87
4.3.2 Gravity waves in the hydrostatic Boussinesq approximation 90
4.3.3 Group velocity and phase speed 92
4.3.4 The structure of gravity waves 93
4.4 Inertial waves 96
4.5 Inertio-gravity waves 97
4.5.1 Dispersion relation 97
4.5.2 Inertio-gravity waves in the hydrostatic Boussinesq approximation 101
4.5.3 Group velocity and phase speed 103
4.5.4 Structure of inertio-gravity waves 104
4.6 Rossby waves 107
4.6.1 Two-dimensional Rossby waves 108
4.6.2 Rossby waves in a stratified fluid 113
4.6.3 Propagation of Rossby waves 116
4.7 Waves on a sphere 118
4.7.1 Shallow water equations and the equation of vertical structure 118
4.7.2 Spherical waves 121

4.7.3 Equatorial waves 127
References and suggested reading 132

5 Instability 133
5.1 Linear stability analysis 133
5.2 Convective instability 135
5.2.1 Rayleigh–Bénard convection 136
5.2.2 Convective instability in a rotating frame. 142
5.3 Inertial instability 145
5.4 Barotropic instability 146
5.4.1 Formulation. 147
5.4.2 Integral theorems 148
5.4.3 Instability of the flow $U = \tanh y$. 150
5.4.4 Interpretation of barotropic instability. 151
5.5 Baroclinic instability. 153
5.5.1 Eady problem 154
5.5.2 Charney problem 167
5.5.3 Energetics 174
5.5.4 Necessary condition for instability 176
5.5.5 Interpretation of baroclinic instability 182
References and suggested reading 184

6 Forced motions 185
6.1 Geostrophic adjustment 185
6.2 Forced motions on the f-plane. 190
6.3 Axisymmetric flows 195
6.4 Forced motions on the β-plane 198
6.5 Forced motions on the equatorial β-plane 199
6.6 Ekman transport 201
References and suggested reading 206

7 Eddy transport 207
7.1 Transport due to finite amplitude waves 207
7.1.1 Generalized Lagrangian mean 207
7.1.2 Examples of finite amplitude waves 209
7.2 Diffusion in the meridional section 216
7.2.1 Governing equations and the Eulerian mean 216
7.2.2 Tracer transport 217
7.3 Residual circulation 221
7.4 Transformed Eulerian mean equations 224
7.4.1 Generalized transformed Eulerian mean equations 224
7.4.2 Quasi-geostrophic TEM equations 227
7.5 Eulerian mean equations in isentropic coordinates 230
7.6 Appendix: Derivation of the generalized Eliassen–Palm relation . 233
References and suggested reading 235

8 Thermodynamics of moist air 236
8.1 Formulation 236
8.1.1 Definition of moist air 236
8.1.2 Basic thermodynamic equations 237
8.1.3 Thermodynamic functions of a single component 240
8.1.4 Mixing of ideal gases 241
8.1.5 Thermodynamic variables of liquid phase 244
8.1.6 Thermodynamic variables of moist air 247
8.1.7 Moist adiabat 249
8.2 Expressions of thermodynamic variables 252
8.2.1 Constants 252
8.2.2 Mass concentrations and saturation condition 254
8.2.3 Thermodynamic functions 256
8.2.4 Moist adiabat 258
References and suggested reading 261

9 Basic equations of moist air 262
9.1 Conservation of mass variables 262
9.2 Conservation of momentum 265
9.3 Conservation of energy and entropy 266
9.4 Transport process 268
9.5 Approximate equations of moist air 269
References and suggested reading 272

10 Radiation process 273
10.1 Blackbody radiation 273
10.2 Solar radiation and planetary radiation 275
10.3 Absorption bands 275
10.4 Radiative transfer equation 277
10.5 Infrared radiation in a plane parallel atmosphere 280
10.6 Gray radiation 283
10.7 Radiative transfer equation of solar radiation 285
References and suggested reading 289

11 Turbulence 290
11.1 Similarity theory 290
11.1.1 Three-dimensional turbulence 290
11.1.2 Two-dimensional turbulance 292
11.1.3 Tracer spectrum 296
11.2 Turbulence models 299
11.2.1 Basic equations 299
11.2.2 Eddy diffusion coefficients and turbulence models 301
11.2.3 The Mellor and Yamada model 304
11.3 Boundary layer 307
11.3.1 Structure of the planetary boundary layer 307

11.3.2 Surface layer 308
11.3.3 Bulk method 309
11.3.4 Boundary layer models 311
References and suggested reading 318

Part II Atmospheric structures 319

12 Global energy budget 322
12.1 Energy budget 322
12.1.1 Effective temperature and global radiative equilibrium . . 322
12.1.2 Energy conversion 325
12.1.3 Total potential energy 330
12.1.4 Available potential energy 334
12.2 Energy budget of a moist atmosphere 336
12.3 Entropy budget and thermal efficiency 338
12.4 Similarity theory of general circulation 342
References and suggested reading 346

13 Latitudinal energy balance 348
13.1 Energy balance model 348
13.2 Latitudinal distribution of radiative balance 350
13.3 Latitudinal temperature difference 354
13.4 Ice albedo feedback 357
13.5 Water budget 361
References and suggested reading 363

14 Vertical structure 365
14.1 Vertically one-dimensional energy balance 366
14.2 Greenhouse effect 368
14.3 Radiative equilibrium 369
14.3.1 Gray radiative equilibrium 369
14.3.2 Static stability of gray radiative equilibrium 373
14.3.3 Approximations of radiative transfer 374
14.4 Radiative–convective equilibrium 376
14.4.1 Mixing length theory 377
14.4.2 Convective adjustment 380
14.5 Radiative–convective equilibrium in a moist atmosphere 383
14.5.1 Energy balance 383
14.5.2 Convective adjustment 384
14.6 Runaway greenhouse effect 386
References and suggested reading 388

15 Moist convection 390
15.1 Circulation structures of convection 391
15.2 Static stability of a moist atmosphere 396

15.2.1 Conditional instability 396
15.2.2 Convective available potential energy: CAPE 397
15.2.3 Saturation condition in the direction of vertical motion . 399
15.3 Circulation structure of moist convection 401
15.3.1 Thermal structure 401
15.3.2 Mass flux 406
15.3.3 Implications for improved cumulus models. 408
15.4 Mixed layer 409
References and suggested reading 413

16 Low-latitude circulations 415
16.1 Dynamics of Walker circulation 416
16.1.1 Large-scale circulation as an organized moist convection 416
16.1.2 Large-scale circulation in a nonrotating frame 419
16.2 Dynamic of Hadley circulation. 421
16.2.1 Cyclostrophic balance 422
16.2.2 Large-scale circulation in a rotating system 423
16.2.3 The Held and Hou model. 429
References and suggested reading 434

17 Circulations on a sphere 436
17.1 Shallow water equations on a sphere. 437
17.2 The Hadley cell model 440
17.3 Mid-latitude circulations 445
17.3.1 Momentum balance of barotropic flow 446
17.3.2 Weak nonlinear theory of Rossby waves 448
17.3.3 Propagation of Rossby waves: WKBJ theory 450
17.3.4 Latitudinal propagation of Rossby waves 453
17.3.5 Angular momentum change 455
17.3.6 Barotropic instability 456
17.4 Turbulence on a sphere 458
17.4.1 Turbulence on a β-plane 458
17.4.2 Two-dimensional turbulence on a sphere 462
17.5 Appendix: Expressions of friction terms on a sphere. 465
References and suggested reading 467

18 Mid-latitude circulations 469
18.1 Meridional circulation 469
18.1.1 Eulerian mean circulation 469
18.1.2 Angular momentum balance between the Hadley cell and the Ferrel cell 472
18.1.3 Transformed Eulerian mean circulation 476
18.1.4 Isentropic mean circulation 478
18.2 Meridional thermal structure 483
18.2.1 Stability of a zonal symmetric state 483

18.2.2 Baroclinic adjustment 485
18.2.3 Tropopause height 487
18.3 Life cycle experiments of extratropical cyclones 491
References and suggested reading 498

19 Global mixing 500
19.1 Potential vorticity and potential temperature 500
19.2 Lagrangian circulation 503
19.3 Stratosphere Troposphere Exchange (STE) 506
References and suggested reading 509

Part III General circulation modelling 511

20 Basic equations of general circulation models 514
20.1 Overview 514
20.2 Basic equations 516
20.2.1 Primitive equations 516
20.2.2 Alternative forms of the equations of motion 518
20.2.3 The equation for sigma velocity 519
20.2.4 The thermodynamic equation 520
20.2.5 Vorticity and divergence equations 521
20.2.6 The moisture equation 523
20.2.7 Summary of governing equations 524
References and suggested reading 525

21 Spectral method on a sphere 526
21.1 The spectrum method 526
21.2 Spectral expansion 526
21.2.1 Spherical harmonics 526
21.2.2 Spectral expansion by spherical harmonics 532
21.2.3 Truncation of spectral expansion 533
21.3 Quadrature on a sphere 535
21.3.1 Gauss–Legendre method 535
21.3.2 Gauss trapezoidal quadrature 538
21.4 Spectral integration method 539
21.4.1 Nondivergent barotropic equation 539
21.4.2 Interaction coefficients method 541
21.4.3 Transform method 541
21.4.4 Procedure of the transform method 544
21.4.5 Grid number of pentagonal truncation 546
21.4.6 Aliasing 548
21.5 Conservation laws of the nondivergent barotropic equation 548
21.5.1 Vorticity 549
21.5.2 Angular momentum 549
21.5.3 Kinetic energy 551

21.5.4 Enstrophy 552
21.6 Shallow water spectral model 553
21.7 Primitive equation spectral model 556
21.7.1 Equations of spectral models 556
21.7.2 Horizontal diffusion and conservation 558
21.8 Appendix A: Associated Legendre functions 559
21.9 Appendix B: Gaussian weights 563
References and suggested reading 566

22 Vertical discretization 567
22.1 Conservation of primitive equations 567
22.1.1 Conservation of mass 568
22.1.2 Constraint on the vertical integral of pressure gradient . . 569
22.1.3 Conservation of total energy 569
22.1.4 Conservation of potential temperature 571
22.2 Conservation of vertical discretization equations 572
22.2.1 Conservation of mass 572
22.2.2 Transformation to the flux form 574
22.2.3 Constraint on the vertical integral of the pressure gradient 576
22.2.4 Hydrostatic balance 578
22.2.5 The thermodynamic equation 583
22.3 Summary 584
References and suggested reading 586

23 Time integration 587
23.1 Time integration scheme and stability 587
23.1.1 Wave equation 588
23.1.2 Damping equation 590
23.1.3 Time filter 591
23.2 The semi-implicit scheme 591
23.2.1 Stability analysis 592
23.2.2 The shallow water model 593
23.3 The semi-implicit method for the primitive equation model 595
23.3.1 Introduction of the semi-implicit scheme 595
23.3.2 Matrix expression of vertical discretization 597
23.3.3 The method for solving the semi-implicit scheme 599
References and suggested reading 602

24 Standard experiments of atmospheric general circulation models 603
24.1 Life cycle of extratropical cyclones experiment 604
24.2 Held and Suarez experiment 606
24.3 Aqua-planet experiment 610
24.4 Atmospheric Model Intercomparison Project (AMIP) 614
References and suggested reading 614

Appendix A1 Transformation of coordinates 617
A1.1 General transformation formulas 618
A1.2 Transformation of orthogonal coordinates 620
A1.3 Basic equations in orthogonal coordinates 624
A1.4 Cylindrical coordinates 625
A1.5 Spherical coordinates 628
References and suggested reading 632

Appendix A2 Physical constants 633
References 634

Appendix A3 Meridional structure 635

Index 640